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FLOW STUDIES OF AXISYMMETRIC BODIES
AT EXTREME ANGLES OF ATTACK

by

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ABSTRACT:

Results of a background investigation and initial theoretical models are described for the flow about axisymmetric bodies at extreme angles of attack. Present state-of-the-art is described with the general conclusions that the current understanding of extreme angle of attack flow phenomena is relatively well developed for angles of attack up to about 40° (symmetric vortex pair in the wake) and that for higher angles of attack there are several important questions yet to be answered. The initial analytical model qualitatively predicts the occurrence of induced yaw forces and is suggested as a basis for further refinements and experimental comparisons. Expressions for the calculation of local forces and pressure distributions are developed for the case of an asymmetric steady vortex wake.

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INTRODUCTION

Operational requirements for air-to-air combat have led to the specification of greatly expanded launch envelopes for guided missiles, where expansion of the launch envelope has been interpreted as increased maneuverability. Thus, regardless of the mode of missile control, flight at extremely high angles of attack is very likely to be required in many encounter situations. The general level of knowledge and understanding of the flow phenomena occurring at extremely high angles of attack (greater than 40 degrees) does not presently allow reliable and systematic prediction of the forces acting on even the most elementary body geometries. Difficulties arise primarily as a consequence of significant yaw forces and the interactions of body vortices as the angle of attack is increased. The present investigation was undertaken in order to critically examine the flow field about axisymmetric bodies at extremely high angles of attack. A qualitative description of the flow field and a prediction model, suitable for calculating body forces and moments, are the desired objectives of this study.

This report describes the results of initial investigations undertaken to establish the existing state-of-the-art. A preliminary analytical model, based upon current understanding of the flow processes, is presented. The continuing investigation is designed to verify the model and to provide refinements to the theory based upon experimental evidence and increases in analytical insight.

BACKGROUND

HISTORICAL SKETCH

Investigations of the behavior of the flow about axisymmetric bodies at angle of attack date back to the simplistic potential flow model of Munk (1924)[1] for the flow over airship hulls. The original models simply accounted for a variable geometry in crossflow planes. Tsien (1938)[2] showed that the potential theory was also applicable to supersonic flight at small angles of attack (less than about 5 degrees). While these potential flow models predicted a normal force distribution and a resulting pitching moment, they lacked completeness in describing the true character of the flow and agreement with experimental data was poor.

A change in the method of analysis was proposed by Jones (1947)[3] and Sears (1948)[4]. They pointed out that theoretically on an infinitely long inclined cylinder with laminar boundary layer flow, the viscous flow across the cylinder may be treated independently of the flow along the cylinder. Thus the component of flow across such an inclined cylinder would be expected to behave as the two-dimensional flow across a cylinder at a velocity equal to the component of the freestream velocity normal to the cylinder (product of the flow velocity past the inclined cylinder and the sine of the angle of inclination). Kuethe (1950)[5] cautioned however that there is no <u>a priori</u> reason to suppose that the locations of transition and turbulent separation on the body are independent of the angle of inclination. Allen (1949)[6] utilized the concept of crossflow independence, postulating that a better evaluation of the normal force distribution on a body of revolution of finite length could be obtained by superposing the inviscid normal force distribution and an additional crossflow term to account for the viscous effects.

This method of analysis led to the impulsive flow analogy proposed by Allen and Perkins (1951)[7]. Briefly, an analogy between time and distance was proposed because "the development of the crossflow with distance along the body on a long body of constant diameter behaves much the same as the development with time of the flow on a circular cylinder impulsively set in motion from rest."[7] They also presented the first qualitative description of wake vortices discharged on the lee side of an inclined body of revolution. It was shown that as the angle of attack increased the vortex system viewed in cross-flow planes changes from a steady symmetric pair (Figure 1a) to a steady asymmetric configuration of two or more vortices (Figure 1b) and, finally, at large angles of attack, to an unsteady asymmetric vortex arrangement (Figure 1c). This same behavior is experienced while accelerating a normal cylinder; that is, a symmetric pair of vortices, the Kármán vortex street, and then the unsteady asymmetric vortex wake configurations.

The awareness and qualitative description of trailing vortices in the wake of inclined bodies stimulated two important experimental investigations. Letko (1953)[8] conducted an experimental investigation of axisymmetric bodies at low speeds and at what were then thought of as large angles of attack (up to 40 degrees). He recorded the existence of side forces and yawing moments with increasing angle of attack due to asymmetrical disposition of trailing vortices. His data indicates a very significant alteration in the yawing moment just below his upper limit of angle of attack. Gowen and Perkins (1953)[9] followed with a study of axisymmetric bodies in supersonic flows. Their efforts were primarily concerned with the possible interaction of the body vortices and tail surfaces for missiles at high angles of attack. They developed the vapor screen technique to visually display the vortex trajectories, and emphasized the factors which affect the onset of unsteady wake flow. Although Gowen and Perkins did not make side force measurements, they did recognize significant variations in the circumferential pressure distributions with changes in the vortex pattern brought about by increasing angle of attack.

At this point the basic qualitative description of the body vortex system and its consequences, as understood today, had been formulated. Techniques of analysis were yet to be developed and a number of alternate approaches were pursued. Kelly (1954)[10] critically re-examined the method developed by Allen and Perkins (1951)[7] of superposing the potential flow and viscous crossflow. Kelly contributed minor modifications to the potential theory, and suggested that the two-dimensional drag coefficient used to account for viscous effects should in fact be derived from transient data since the crossflow is time dependent. These modifications improved agreement between experimental data and calculated results for slender axisymmetric bodies at angles of attack of less than 25 degrees. Thus, although the prediction capability had been improved, it did not extend into the angle of attack range at which asymmetric vortex shedding may be most significant with respect to side forces and yawing moments.

Early attempts at modeling the wake vortex system were hampered by inadequate visual confirmation. Maltby and Peckham (1956)[11] developed a new smoke technique to visually display the vortex pattern above inclined slender bodies. Their smoke technique is similar in principle to the vapor screen technique used by Gowen and Perkins (1953)[9] in supersonic flow, except it was applicable to low speed flows.

Among the earliest attempts to model the wake vortex characteristics of an inclined axisymmetric body were those of Perkins and Jorgensen (1956)[12], Jorgensen and Perkins (1958)[13], and Tinling and Allen (1962)[14]. This series of reports constitutes an extensive study of the flow about an inclined ogive-cylinder body in both subsonic and supersonic flow. To aid in their understanding of the experimental results, they developed a model in which the induced flow field in any crossflow plane along the cylindrical afterbody is represented by the incompressible flow around a circular cylinder in the presence of a symmetric pair of vortices. They achieved reasonably good agreement between the experimentally observed and calculated vortex trajectories using the experimentally derived vortex starting positions, vortex strengths, and normal force distributions. Their model is noteworthy in that it illustrated the possible utility of modeling the viscous crossflow via the potential flow about a normal cylinder in the presence of discrete regions of concentrated vorticity. Their experimental data represents one of the first rigorous mappings of the wake vortices and their strengths. Mello (1959)[15] utilized a similar potential flow analysis for the flow about a normal cylinder in the presence of symmetrical vortices and was able to confirm that the strengths of the discrete vortices in the body vortex wake could be estimated provided the viscous normal force distribution and the vortex locations were known.

The importance of related detailed investigations of flow phenomena such as the two dimensional potential and boundary layer flow over cylinders in steady and time dependent flow, boundary layer transition and separation, vortex feeding regions, and wake vortex motions became more apparent. The extensive treatments of Roshko (1953),[16] (1954),[17] (1955),[18] (1960)[19] of the flow about cylinders was a significant aid in understanding boundary layer transition, separation, and wake formation associated with the cylinder in the crossflow plane over a wide range of flow parameters. Bryson (1959)[20] developed a model for the wake development behind a cylinder by considering a separated pair of symmetrical vortices joined to the body by straight feeding sheets. Using the flow analogy in which the flow in a crossflow plane at a distance z from the nose is analogous to the cylinder crossflow at time $t = z/V_0 \cos\alpha$, Bryson gave solutions for the vortex strengths and positions as functions of time (or distance along the body) and calculated the resulting normal force distributions for circular cones and cylinders. This model of a time varying vortex wake behind a circular cylinder did, however, require the specification of the point at which the feeding sheet departs from the body and the position of the vortices at the instant when the calculations begin.

Additional insights into the time dependent characteristics of the flow about circular cylinders in a crossflow plane were presented by Sarpkaya (1963)[21], Sarpkaya and Garrison (1963)[22], and Sarpkaya (1966)[23]. These investigations contributed detailed analysis and experimental data on the time dependent body forces and associated wake formation, noting a significant rise in the normal force coefficient above its steady state value during the growth of the vortices. For the inclined axisymmetric body the time dependent features can again be related to a crossflow plane at a fixed axial location.

Schindel (1963)[24] (1965)[25], Friberg (1965)[26] (1965)[27], and Schindel and Chamberlain (1967)[28] provided a unified analysis and experimental verification of the vortex separation on axisymmetric inclined bodies of general elliptic cross section utilizing the Bryson (1959)[20] model of the vortex feeding sheet. Excellent agreement of theory with experimental data of the normal force coefficient, center of pressure, separation angle, and vortex trajectories was obtained for an angle of attack range of 0 to 30 degrees and for numerous nose-body configurations.

Most of the modeling of the wake vortices for inclined axisymmetric bodies up to this time had assumed a symmetric pair of rolled up vortex sheets extending the entire length of the body (Figure 2). This model has been termed the "NACA vortex model," and in general is appropriate for relatively short bodies or at relatively low angles of attack. It has already been noted that as the angle of attack increases the wake vortex pattern in the crossflow planes changes from that of a symmetric pair to a steady asymmetric configuration of two or more vortices. In a crossflow plane, the steady asymmetric shedding of vortices is analogous to the well-known Kármán vortex street. Thomson and Morrison (1965)[29] postulated a theoretical model for describing the asymmetric shedding of vortices from slender cylindrical bodies at large angles of inclination. Their model utilizes the strengths and positions of the vortices in a crossflow plane obtained from Kármán vortex street theory, in conjunction with the analogy proposed by Allen and Perkins (1951)[7]. This method of analysis yields vortex strengths, vortex trajectories, and axial spacing of the vortex origins if the point of departure of the first trailing vortex is prescribed (Thomson and Morrison assumed the first vortex was shed at the shoulder between the nose and afterbody). Limited experimental verification is included, with the exception of vortex spacing. Numerous Schlieren photographs of a cone-cylinder body at various angles of attack vividly display the existence of multiple trailing vortices.

Use of the Kármán vortex street theory requires an initial input of either the strength of the vortices or a force coefficient. Thus, in an effort to enhance the proposed theory using a Kármán vortex street, Thomson (1966)[30] developed a model of the two-dimensional flow past a circular cylinder in the subcritical Reynolds number range from which vortices are shed periodically. The model provides a method of determining the strength of the shed vortices by considering the vorticity generated in the boundary layer from the forward stagnation point to the point of separation. By Kelvin's theorem, the circulation in any particular shed vortex is the same as that which has been built up around the boundary layer just prior to vortex shedding. The theoretical vortex strength and experimentally determined values of the Strouhal number (the non-dimensional frequency at which vortices are shed or at which the forward stagnation point shifts from one extreme to another) are introduced into the Kármán vortex street theory to give an estimate of a mean drag coefficient (normal force on inclined body) and also a mean lift coefficient (yaw on an inclined body). Comparison with experimental values of the lift and drag of a two-dimensional cylinder for subcritical Reynolds numbers is quite good. The range of subcritical Reynolds numbers includes 1×10^2 as a lower limit due to viscous domination, and about 1×10^5 as an upper limit due to boundary layer transition.

The vortex feeding region was again studied by Sarpkaya (1968)[31] in an effort to reconcile some of the difficulties of the single line vortex feeding sheet of Bryson (1959)[20]. Sarpkaya proposed a feeding zone of an infinite number of two dimensional vortex sheets, connected to the nascent vortices, and feeding them until they acquire sufficient strength and are convected downstream while other small discrete vortices are formed adjacent to the feeding zone. Presuming that the nascent vortex develops from the point of physical boundary layer separation on a circular cylinder, Sarpkaya was able to calculate the normal force coefficient and the circulation, which were in close agreement with those observed experimentally. The application of this vortex formation model to the flow in a crossflow plane of an axisymmetric inclined body is a logical extension.

The earlier ideas of Thomson and Morrison (1965)[29] and Thomson (1966)[30] were unified and presented in Thomson and Morrison (1969)[32] (1971)[33]. Their theories provide methods of calculating sectional body forces and the spacing, position, and strength of vortices in the wake of slender cylindrical bodies at large incidence, provided the wake vortices behave as a Kármán vortex street in crossflow planes. In order to calculate the four variables mentioned, one must be empirically specified. The simplicity of the model is astonishing when considered in light of its experimentally proven validity. Experimental data was compiled for subcritical Reynolds numbers, for both incompressible and compressible flow, and for angles of attack up to 40 degrees.

Fletcher (1969)[34] (1971)[35] (1971)[36] contributed invaluable theoretical and experimental data on the Magnus characteristics of a spinning inclined ogive cylinder body at subcritical Reynolds numbers in incompressible flow. Although his primary interest was the Magnus characteristics, data are presented for zero spin. In modeling the flow about a spinning circular cross-section, Fletcher could not assume a Kármán vortex street due to the dislocation of the vortices as a result of spin. His analysis was therefore based upon the potential flow about a circular cross-section with circulation and in the presence of two discrete vortices. Valuable experimental data is presented on vortex spacing and position, normal and yaw force coefficients, pitch and yaw moments, and separation points for various angles of attack up to 30 degrees and for spin ratios between zero and one. Fletcher also presented a novel experimental technique for locating the point of boundary layer separation using a hot wire anemometer.

Recent efforts at the Naval Ship Research and Development Center by Saffel, et al. (1971)[37], Krouse (1971)[38] and Pick (1971)[39] have contributed to the design aspects of axisymmetric bodies at large angles of attack. Various nose geometries, nose-body configurations, and body-fin arrangements were investigated. The work of Saffel, et al. (1971)[37] constitutes an automated method for predicting the static aerodynamic characteristics of complete missile configurations at large angles of attack, relying almost exclusively upon empirical input. Pick's (1971)[39] presentation emphasizes the significance of the nose geometry in establishment of the wake flow conditions over the afterbody.

SUMMARY OF DEVELOPMENTS

In summarizing the historical development of the methods of analysis and experimental evidence for the flow about inclined axisymmetric bodies, two questions are appropriate:

(1) What flow and force parameters can be obtained from the various theories?

(2) What are the limitations of the present theories?

Experimental evidence proves the validity of the qualitative description originally proposed by Allen and Perkins (1951)[7]: the vortex system viewed in crossflow planes changes with increasing angle of attack from a steady symmetric pair, to a steady asymmetric configuration of two or more vortices, to an unsteady asymmetric arrangement. An additional change from the unsteady asymmetric vortex wake to either a completely turbulent wake or a return to a steady asymmetric arrangement should be expected as the angle of attack increases, approaching 90 degrees. Due, however, to a finite body length, they should not be expected to appear as the idealized vortex pattern behind a normal two-dimensional cylinder.

The analytical models each have an appropriate range of applicability, and can be classified primarily by the assumed wake vortex pattern. In general, the theories just reviewed do allow the prediction of the aerodynamic coefficients for simple cylindrical bodies at angles of attack up to about 40 degrees. In some cases (Thomson and Morrison (1971)[33]) the complete integration of body forces and moments has yet to be completed. The actual structure of the wake vortex system is either assumed or predicted by the various models. Within the angle of attack range 0 to 40 degrees, three main features remain as important difficulties: (1) prediction of boundary layer separation on a three-dimensional inclined body, (2) methods of identifying analytically the origin of the first vortex trail, and (3) the effects of supercritical crossflow Reynolds numbers on the development and disposition of wake vortices. The addition of an integrated model for the prediction of the aerodynamic coefficients, in the angle of attack range in which steady asymmetric vortex shedding occurs, remains for the completion of the predictive model for angles of attack of up to about 40 degrees.

For angles of attack above about 40 degrees theoretical work is nearly nonexistent. Recent experimental evidence does indicate the existence of significant yaw forces and moments, often exceeding the control capabilities of the missile, and thus giving justification to the establishment of a prediction capability. These phenomena have been recorded in the angle of attack range of 40 to 60 degrees. Additional information is also required to adequately describe the wake as the angle of attack approaches 90 degrees.

PRELIMINARY ANALYTICAL MODEL

NOMENCLATURE

b	-	radius of circle in the z plane
D	-	diameter of circle in the z plane
$\bar{F}$	-	complex conjugate force
f	-	shedding frequency of vortices of like sign
G_k	-	non-dimensional vortex strength (Γ_k/UD)
g	-	axial spacing of shed vortices
h	-	transverse spacing of vortices in Kármán vortex street
ℓ	-	lateral spacing of vortices of like sign in Kármán vortex street
P_b	-	pressure on body
P_∞	-	freestream pressure
Re_c	-	crossflow Reynolds number ($\frac{V_o D \sin\alpha}{\nu}$)
S	-	Strouhal number (Eq. (1))
S_k	-	relative complex position vector of vortex (z_k/b)
U	-	normal component of velocity ($V_o \sin\alpha$)
V_o	-	freestream velocity
X	-	real component of complex force (in the yaw direction)
x	-	real coordinate in complex z plane
Y	-	imaginary component of complex force (in the pitch direction)
y	-	imaginary coordinate in complex z plane
z	-	complex coordinate, $x + iy$
α	-	angle of attack relative to freestream direction vector
Γ	-	strength of vortex, circulation
θ	-	polar angle in complex z plane

ν - fluid kinematic viscosity

ξ - angle between vortex trail and body axis

ρ - fluid density

ρ_k - radial coordinate of vortex

ϕ_k - polar angle of vortex

Ω - complex velocity potential

DISTRIBUTION AND STRENGTH OF TRAILING VORTICES

The description of the flow about axisymmetric bodies at angle of attack, based upon the flow analogy proposed by Allen and Perkins (1951)[7] and utilized in conjunction with Kármán vortex street theory by Thomson and Morrison (1969)[32] (1971)[33], provides the starting point for the following discussion.

Consider the flow about an inclined axisymmetric body illustrated in Figure 3. In the sectional planes the flow pattern resembles that of a two-dimensional wake behind a normal circular cylinder. According to the flow analogy, the development of the flow in the crossflow planes with time is analogous to the flow development with position along the body axis. Along the nose section, the flow in the crossflow planes is accelerating as a result of the axially varying geometry, and corresponds to the development of the flow about a two-dimensional circular cylinder impulsively set in motion from rest. The accelerating flow causes an increase in the crossflow Reynolds number. The steady two-dimensional wake flow behind normal circular cylinders, as a function of Reynolds number, has been extensively investigated. Briefly, at very low Reynolds numbers the viscous action is so great in comparison with the inertia of the fluid that separation does not occur. At higher Reynolds numbers (of the order 50) a symmetric vortex pair forms. At still higher Reynolds numbers (50 to 3×10^5) asymmetry and alternate detachment of wake vortices occurs. In their early states of formation the vortex trails display the geometric characteristics of the stable potential flow patterns analyzed by Kármán. At even higher Reynolds numbers (greater than 3×10^5) the boundary layer on the cylinder becomes turbulent prior to separation and the wake ceases to be periodic.

The flow in the crossflow planes develops with increasing crossflow Reynolds number along the nose section, reaching a steady state value of the crossflow Reynolds number at the junction of the nose and cylindrical afterbody. The steady state value of the crossflow Reynolds number should correspond to a particular wake vortex pattern on the afterbody. The wake vortex pattern and the crossflow Reynolds number do not necessarily correlate however, since the nose geometry or the freestream flow conditions may establish some alternate pattern. Momentarily, assume that the nose geometry, freestream conditions and angle of inclination are such that a laminar boundary layer and periodic vortex shedding does exist in the crossflow planes. Then, Kármán vortex street theory provides a basis for estimating the strength and path of the vortices as well as the position at which each vortex is shed from the body. A knowledge of the vortex strengths and positions can be utilized with a two-dimensional hydrodynamic model to calculate the sectional forces. Integration of the sectional forces over the length of the body will yield the body force and moment coefficients. The following discussion will illustrate this technique, but it should be emphasized that significant problems are as yet unanswered, principally with regard to the nose section and the origin of the first vortex.

With reference to Figure 3, the velocity vector is decomposed into a normal component, $V_o \sin\alpha$, and an axial component, $V_o \cos\alpha$. At any section the Strouhal number, a measure of the frequency of vortex shedding relative to the size of the body and the speed of the freestream can be written as

$$S = \frac{fD}{V_o \sin\alpha} \tag{1}$$

The Strouhal number varies with Reynolds number, but for a rigid right circular cylinder maintains a magnitude of about 0.2 over a wide Reynolds number range ($1 \times 10^3 < Re < 1 \times 10^5$), in particular within the Reynolds number range for which the Kármán vortex street occurs. Therefore, assuming the Strouhal number is known and constant, then a fluid particle A (see Figure 3) on the upstream generatrix of the body will contribute to the vortex of like sign, formed a distance 2g downstream. The time required for the fluid particle A to traverse the required distance is given by

$$\frac{1}{f} = \frac{D}{S\, V_o \sin\alpha} \tag{2}$$

But the velocity with which the particle A is carried along the axis is $V_o \cos\alpha$, so

$$V_o \cos\alpha = \frac{2g}{1/f} \tag{3}$$

and combining

$$2g = \frac{D \cot\alpha}{S} \tag{4}$$

thus, in the Reynolds number range where the vortex shedding is periodic, the spacing between shed vortices can be estimated.

The axial position at which the first vortex is shed, as noted earlier, is effected by such things as nose geometry and fluctuations in the free-stream conditions. Proceeding along the axis of the nose the Reynolds number is increasing from a very small value near the tip, and either separation does not occur or a symmetrical vortex pair is formed through separation on both sides. Continuing along the axis, the symmetrical vortex pair will grow at an ever reducing rate. Eventually, one vortex will tend to overbalance the other and pass off into the wake. As the second continues to become larger, a third will form in the place of the first, and a process of alternate formation and detachment is thus ultimately established. The quantization of this description as applied to the nose section of an axisymmetric body is an area which will be more thoroughly investigated. This will be a necessary step prior to the integration of sectional force calculations to obtain body forces and moments.

Returning to Figure 3, the angle, ξ, that the vortex core makes with the axis of the body can be estimated by noting that

$$\cot\xi = \frac{2g}{\ell} \tag{5}$$

where ℓ is the lateral spacing between vortices of like sign in the Kármán vortex street. Substituting for g yields

$$\cot\xi = \frac{1}{S}\left(\frac{D}{\ell}\right)\cot\alpha \tag{6}$$

that is, the vortex follows a trajectory which is not aligned with the axis of the body nor with the freestream. An important feature to be noted is that the axis of rotation of the vortex is not normal to the crossflow plane. Therefore, an immediate question arises as to the interpretation of circulation, or vortex strength, in the Kármán vortex street model. Methods for resolving this dilemma are being investigated. Temporarily, assume that the angle between the vortex core and the normal to the crossflow plane is small so that Kármán vortex street theory is applicable. Then, the lateral spacing, ℓ, can be deduced from the Kármán street theory which provides that for a stable system

$$\tanh \frac{\pi h}{\ell} = \frac{1}{\sqrt{2}} \tag{7}$$

or

$$\frac{h}{\ell} = 0.281 \qquad (8)^*$$

For laminar separation, h is approximately equal to D. Thus, as an initial estimate assume that

$$\frac{D}{\ell} = 0.28 \tag{9}$$

The strength of the vortices, illustrated in the Kármán vortex street of Figure 1b, is given by the theory as

$$\frac{\Gamma}{UD} = 2\sqrt{2}\left(\frac{\ell}{D}\right)\left[1 - S\left(\frac{\ell}{D}\right)\right] \tag{10}$$

At this point enough information is available to describe the wake pattern at any section along the axis of an axisymmetric body at angle of attack, exclusive of the origin of the first vortex. The major difficulties thus far include: (1) the origin of the first vortex, (2) the contribution of the wake formation on the nose, and (3) the assumptions concerning the lateral spacing of a stable vortex system.

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*Thomson and Morrison (1971)[33] reported values of h/ℓ as low as 0.19 for a cone-cylinder body.

WAKE INDUCED FORCES

The sectional forces acting on a stationary circle in a uniform flow in the presence of n frozen vortices can be calculated by the techniques of theoretical hydrodynamics, namely the steady form of the Blasius Theorem,

$$\bar{F} = X - iY = \frac{i\rho}{2} \oint \left(\frac{d\Omega}{dz}\right)^2 dz \tag{11}$$

where X and Y are the components of the force acting in the complex z plane, and Ω is the complex velocity potential using the sign convention $d\Omega/dz = u - iv$. The line integral is evaluated about the body contour.

Consider a circle in a uniform flow directed along the positive y axis and in the presence of n discrete vortices as illustrated in Figure 4. In the absence of the circle the complex potential is

$$f(z) = -iUz - \frac{i}{2\pi} \sum_{k=1}^{n} \Gamma_k \ln(z - z_k) \tag{12}$$

where the sign of a particular vortex is associated with Γ_k (counterclockwise being positive). Application of the Circle Theorem yields the complex velocity potential for the flow about the circle in the presence of the uniform flow and the n vortices, that is

$$\begin{aligned} \Omega(z) = i \Bigg[U \left(\frac{b^2}{z} - z\right) &- \frac{1}{2\pi} \sum_{k=1}^{n} \Gamma_k \ln(z - z_k) \\ &- \frac{1}{2\pi} \ln z \sum_{k=1}^{n} \Gamma_k + \frac{1}{2\pi} \sum_{k=1}^{n} \Gamma_k \ln\left(z - \frac{b^2}{\bar{z}_k}\right) \\ &+ \cdot \cdot \cdot \Bigg] \end{aligned} \tag{13}$$

where the terms omitted are strictly constants and will therefore not contribute to the force. The three summed terms represent the contributions to Ω from the parent vortices and their images in the circle, while the first term represents a uniform flow about the circle. Noting that z_k is outside the contour of integration (the body contour) and performing the indicated operations, the resultant sectional force on the circle is given by

$$\bar{F} = X - iY = -\rho U \sum_{k=1}^{n} \Gamma_k \left(\frac{b}{z_k}\right)^2 + \frac{\rho}{2\pi} \sum_{j=1}^{n} \sum_{k=1}^{n} \Gamma_j \Gamma_k \left[\frac{b^2}{z_k(z_k \bar{z}_j - b^2)}\right] \tag{14}$$

The first term on the right is a contribution due to the images of the parent vortices yielding a term as would be expected for a circle with circulation. The second term on the right represents a contribution arising from the interaction between parent vortices and between parent vortices and their images in the circle. In general, both terms have real and imaginary parts, giving rise to both X and Y components of force. Non-dimensionalizing Eq. (14) with respect to the freestream dynamic pressure and the circle diameter, the sectional force coefficients become

$$\frac{\bar{F}}{1/2\rho U^2 D} = C_{Fx} - iC_{Fy} = -2 \sum_{k=1}^{n} G_k \left(\frac{1}{S_k}\right)^2 + \frac{2}{\pi} \sum_{j=1}^{n} \sum_{k=1}^{n} G_j G_k \left[\frac{1}{S_k(S_k \bar{S}_j - 1)}\right] \tag{15}$$

Equation (15) has been programmed (see Appendix A) to facilitate computations.

As an example of the affects of multiple vortices on the sectional force, consider two cases:

Case I

For a circle in the presence of a uniform flow and a single positive vortex, Eq. (14) becomes (n = 1)

$$F = X - iY = -\rho U\Gamma_1 \left(\frac{b}{z_1}\right)^2 + \frac{\rho\Gamma_1^2}{2\pi} \left[\frac{b^2}{z_1(z_1\bar{z}_1 - b^2)}\right] \qquad (16)$$

Case II

For a circle in the presence of a uniform flow and two vortices of opposite sign (Γ_1 positive and Γ_2 negative, Eq. (14) becomes (n = 2)

$$\bar{F} = X - iY = -\rho U\Gamma_1\left(\frac{b}{z_1}\right)^2 + \rho U\Gamma_2\left(\frac{b}{z_2}\right)^2$$

$$+ \frac{\rho\Gamma_1^2}{2\pi}\left[\frac{b^2}{z_1(z_1\bar{z}_1 - b^2)}\right] + \frac{\rho\Gamma_2^2}{2\pi}\left[\frac{b^2}{z_2(z_2\bar{z}_2 - b^2)}\right] \qquad (17)$$

$$- \frac{\rho\Gamma_1\Gamma_2}{2\pi}\left[\frac{b^2}{z_1(z_1\bar{z}_2 - b^2)}\right] - \frac{\rho\Gamma_1\Gamma_2}{2\pi}\left[\frac{b^2}{z_2(z_2\bar{z}_1 - b^2)}\right]$$

A comparison of Eq. (17) with Eq. (16) illustrates the additional terms arising due to the interaction of the vortex of strength Γ_1 and the vortex of strength Γ_2 by the presence of the last two terms in Eq. (17).

The resultant sectional force is most easily illustrated by returning to Eq. (16) and letting $z_1 = iy_1$. That is, the single positive vortex is located on the positive y axis as shown in Figure 5. Then

$$\bar{F}\big|_{z_1 = iy_1} = \rho U\Gamma_1\left(\frac{b}{y_1}\right)^2 - \frac{i\rho\Gamma_1^2}{2\pi}\left[\frac{b^2}{y_1(y_1^2 - b^2)}\right] \qquad (18)$$

Thus, there is a component of force in the direction of the positive x axis (the typical lift on a circle due to circulation), and a component in the direction of the positive y axis. For the inclined axisymmetric body, these components translate to a sectional normal force and an induced sectional yaw force.

Another particularly significant result is obtained for a symmetric vortex pair. Letting $\Gamma_1 = \Gamma_2$ and $z_2 = -\bar{z}_1$ in Eq. (17), it can be shown that only imaginary terms remain for the vortex induced forces. Therefore, for symmetric vortex shedding there is no induced yaw force, that is $X = 0$. This observation leads to a qualitative explanation of the widely observed induced yaw forces occurring at intermediate angles of attack where vortex shedding is not symmetric. The occurrence of asymmetrical vortex patterns in the crossflow planes leads to alternating signs for the induced yaw force along the body axis so that when many vortices are present the integrated body force may appear relatively free of induced yaw effects. For example, consider the asymmetric shedding of vortices of equal strength along the afterbody. If there is an even number of trailing vortices then the net yaw force on the afterbody vanishes, and for the entire body the yaw force would be relatively small, attributed to yaw forces on the nose. The yaw moment, in any case, is still quite significant.

Equation (14) provides a theoretical expression for the sectional forces on a circle in the presence of a uniform flow and n stationary vortices. Within the limitations of incompressible potential flow, the sectional force calculated from Eq. (14) for vortices of arbitrary strength and position is theoretically correct. Although Eq. (14) yields theoretically correct results for vortices of arbitrary strength and position, it must be cautioned that physically meaningful results are not derived from arbitrary placement of the vortices. Vortices of arbitrary strength and position are not necessarily stable in a real flow situation. That is, if a discrete vortex is arbitrarily inserted into the steady potential flow field about a circle, then the vortex will seek a stable position before it comes to rest with respect to the circle. At any position other than the stable position the foregoing steady analysis is invalid. The essence of the above discussion is illustrated in Figure 6. A single vortex was positioned at increasing distances from the circle. Close to the circle ($|S| = 1$) both components of the force increase without bound. As the position is increased the force coefficients decay. At some particular configuration the y component vanishes, then with increasing distance, reaches a maximum negative value and finally approaching zero from the negative side. The x component (yaw force) remains positive and directed toward the vortex. The important feature is that, at least instantaneously, with a particular vortex configuration the body may experience a negative drag (normal force).

The primary utility of the steady flow analysis is the evaluation of the sectional forces on an inclined axisymmetric body in the presence of multiple stationary vortices. It does not provide the necessary information to determine what vortex configurations are stable; stability criterion must be developed from an unsteady analysis. Note, however, that the Kármán vortex street represents a stable vortex configuration, thus strengths and positions derived from this theory provide meaningful inputs to Eq. (14). For the steady symmetric pair, the Föppl curve (illustrated in Figure 1a), representing the locus of points where a symmetric vortex pair can be located at rest with respect to a circle in a steady flow, provides the strengths and positions of stable vortex configurations. The Föppl curve is discussed in Reference 20.

PRESSURE DISTRIBUTIONS

The techniques of theoretical hydrodynamics can also be utilized to calculate the pressure distribution about a body in the presence of discrete singularities. The pressure coefficient, defined as

$$C_p = \frac{P_b - P_\infty}{\frac{1}{2}\rho U^2} = 1 - \frac{\left(\frac{d\Omega}{dz}\right)\left(\overline{\frac{d\Omega}{dz}}\right) \Big|_{|z| = b}}{U^2} \tag{19}$$

has been developed for the multiple vortex configuration illustrated in Figure 4. In a non-dimensional form the pressure coefficient is given by

$$\begin{aligned} C_p = &- (1 + 2\cos 2\theta) \\ &+ \frac{8(\cos\theta \cos 2\theta)}{\pi} \sum_{k=1}^{n} G_k \left[\frac{\left(\frac{\rho_k}{b}\right)\cos(\theta - \phi_k) - 1}{1 + \left(\frac{\rho_k}{b}\right)^2 - 2\left(\frac{\rho_k}{b}\right)\cos(\theta - \phi_k)} \right] \\ &- \frac{4}{\pi^2} \left\{ \sum_{k=1}^{n} G_k \left[\frac{\left(\frac{\rho_k}{b}\right)\cos(\theta - \phi_k) - 1}{1 + \left(\frac{\rho_k}{b}\right)^2 - 2\left(\frac{\rho_k}{b}\right)\cos(\theta - \phi_k)} \right] \right\}^2 \end{aligned} \tag{20}$$

Equation (20) has been programed to facilitate computations (see Appendix B). The pressure distribution about the circle is an indication of the force distribution (potential flow), and provides the pressure impressed upon the boundary layer for boundary layer analysis. Examples of pressure distributions are illustrated in Figure 7 for a single positive vortex, and Figure 8 for a symmetric pair of vortices. In the case for the single vortex (Figure 7) asymmetry of the low pressure region nearer the vortex implies a force in that direction. For the symmetric case (Figure 8) the distribution is symmetric implying no induced yaw force. The force on the body can also be obtained by integrating the pressure coefficient around the body contour, which in essence yields Eq. (14). The shifting of the stagnation points, and the existence of favorable and adverse pressure gradients as a result of the presence of vortices provides information useful in correlating the separation points, and conducting boundary layer analysis.

BODY FORCES AND MOMENTS

Combining the information developed in the previous sections provides a means of describing the sectional forces acting on an inclined axisymmetric body. The integration of the sectional force distributions along the entire length of the body will then provide the body forces and moments. Once the distribution of the sectional forces is available this is a simple task, however a complete description of the sectional forces on the nose has not been developed.

Again, referring to Figure 3, it is readily apparent that, sectionwise, an alternating yaw force may be present. Under certain conditions (an even number of shed vortices) the alternating sectional yaw forces may cancel, however, the yaw moment may still be significant. Therefore, at a particular set of freestream conditions and angle of attack, the existence of a net yaw force will depend upon the length of the body. A yaw moment will always be present if the vortex pattern at any section is asymmetric.

SUMMARY OF PRELIMINARY ANALYTICAL MODEL

It is anticipated that the analytical model can be combined and refined to provide direct output of force and moment coefficients as a function of a minimum number of inputs for the purposes of design and prediction.

Some important problems as yet not fully understood include:

1. The prediction of the origin of the first vortex participating in the periodic asymmetric shedding.

2. A complete description of the sectional forces on the nose portion of the body.

3. The significance of the lateral spacing of the vortices in the street for an inclined cylinder.

4. The true behavior of a vortex whose axis of rotation is skewed to the plane of the vortex street.

5. The permissibility of the application of steady potential flow theory to the vortex dominated wake region.

6. The ranges of angle of attack and freestream Reynolds number in which particular wake flow phenomena (steady symmetrical, steady asymmetric, and unsteady asymmetric vortex patterns) are characteristic of the flow field.

Although the present analytical model provides insight into the simpler aspects of the flow, it is at present only qualitatively useful. A complete predictive model will of necessity require considerable support from the experimental portions of this investigation.

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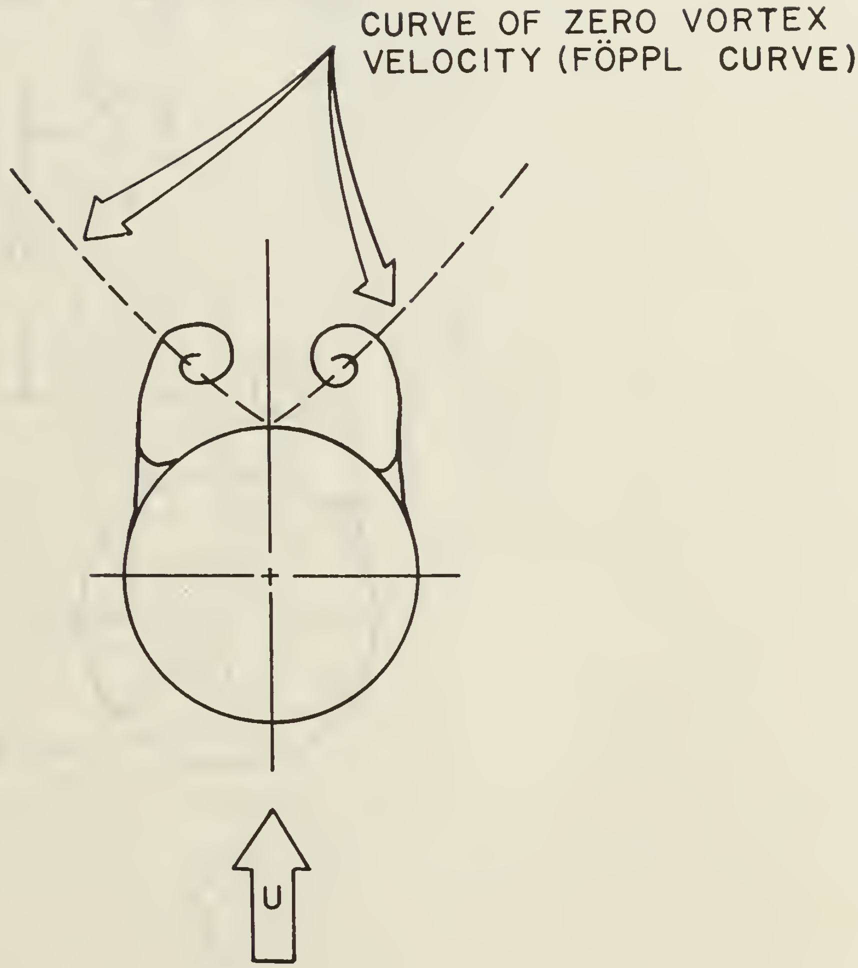

FIGURE 1a: STEADY SYMMETRICAL

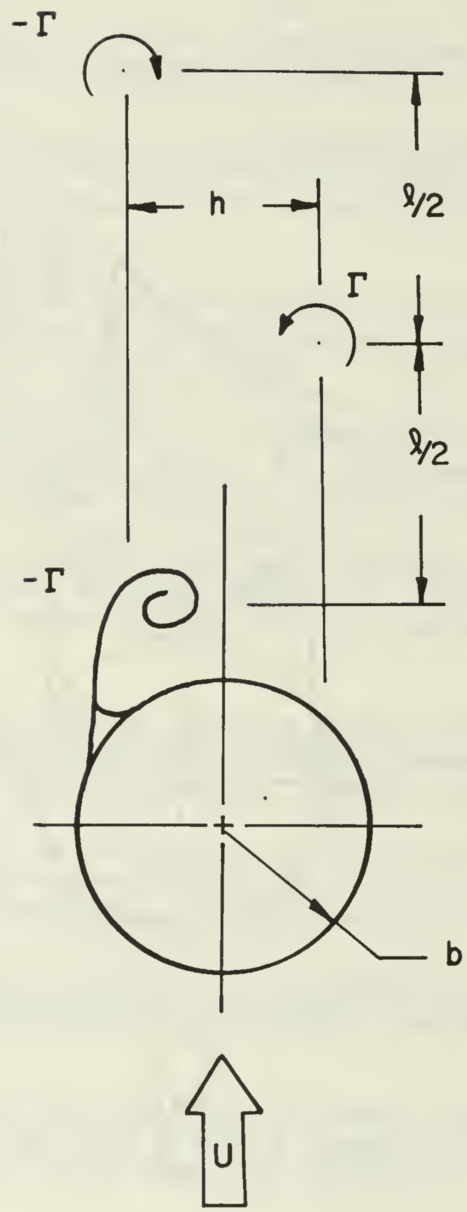

FIGURE 1b: STEADY ASYMMETRIC
(KÁRMÁN VORTEX STREET)

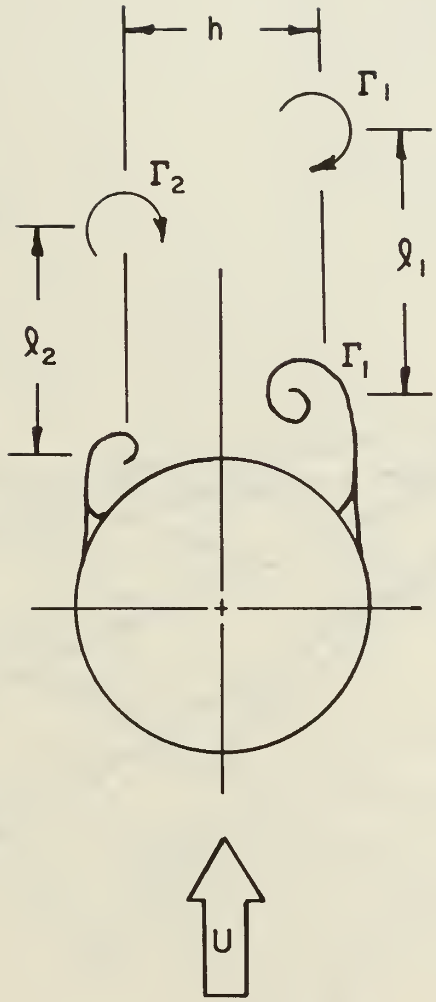

FIGURE 1c: UNSTEADY ASYMMETRICAL

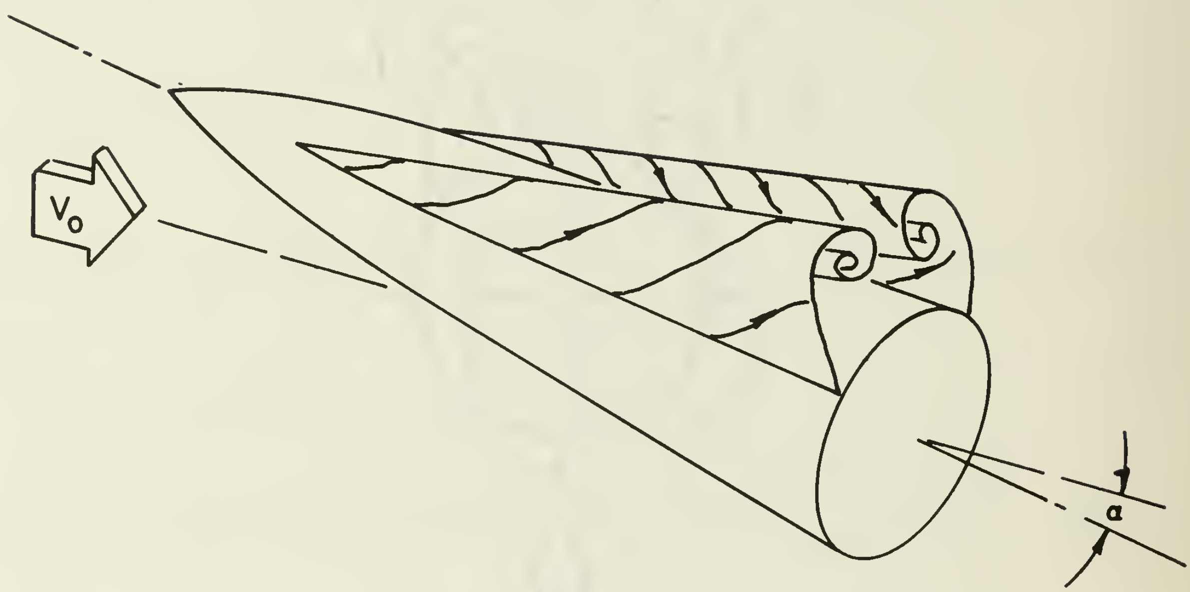

FIGURE 2: NACA VORTEX MODEL

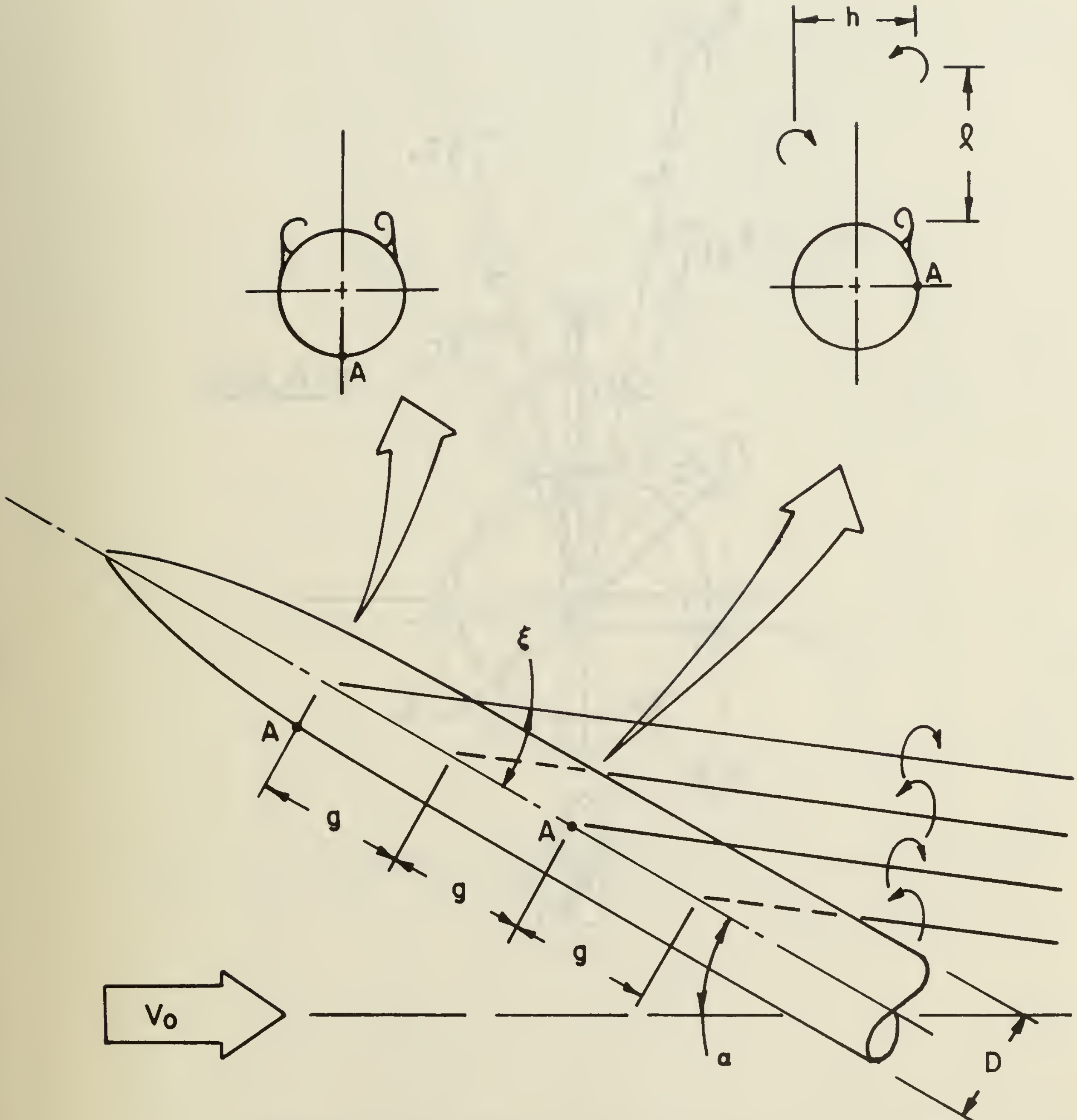

FIGURE 3: SCHEMATIC DIAGRAM OF AXISYMMETRIC BODY AT ANGLE OF ATTACK

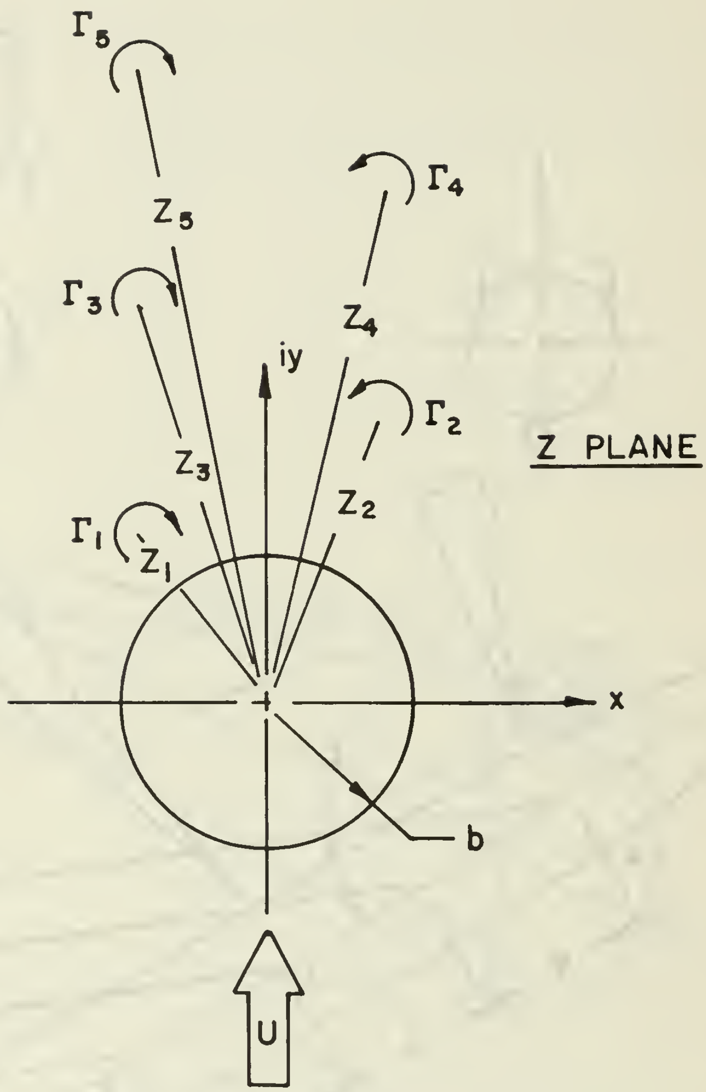

FIGURE 4: COMPLEX Z PLANE FOR UNIFORM FLOW PAST A CIRCLE IN THE PRESENCE OF n VORTICES

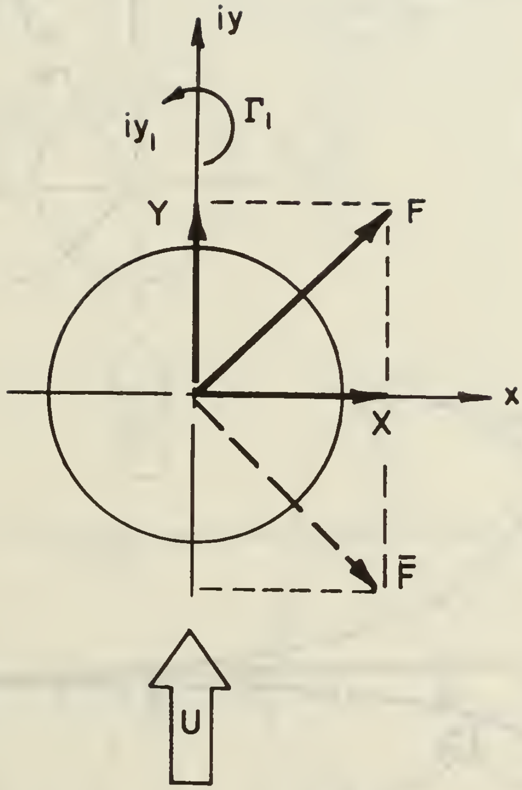

FIGURE 5: SECTIONAL FORCE ON A CIRCLE IN THE PRESENCE OF A UNIFORM FLOW AND A SINGLE VORTEX

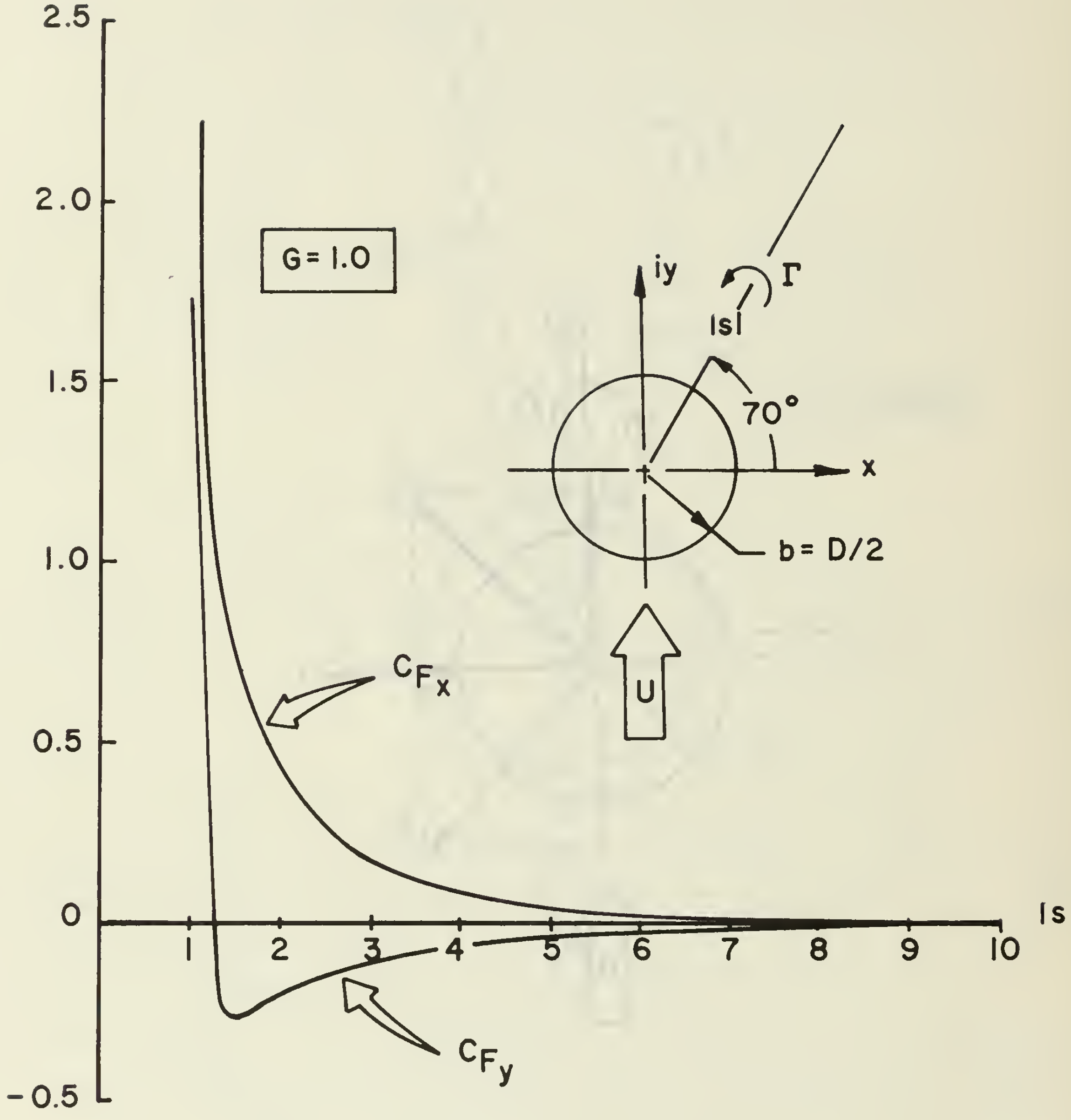

FIGURE 6: VARIATION OF SECTIONAL FORCE COEFFICIENTS FOR SINGLE VORTEX WITH INCREASING POSITION

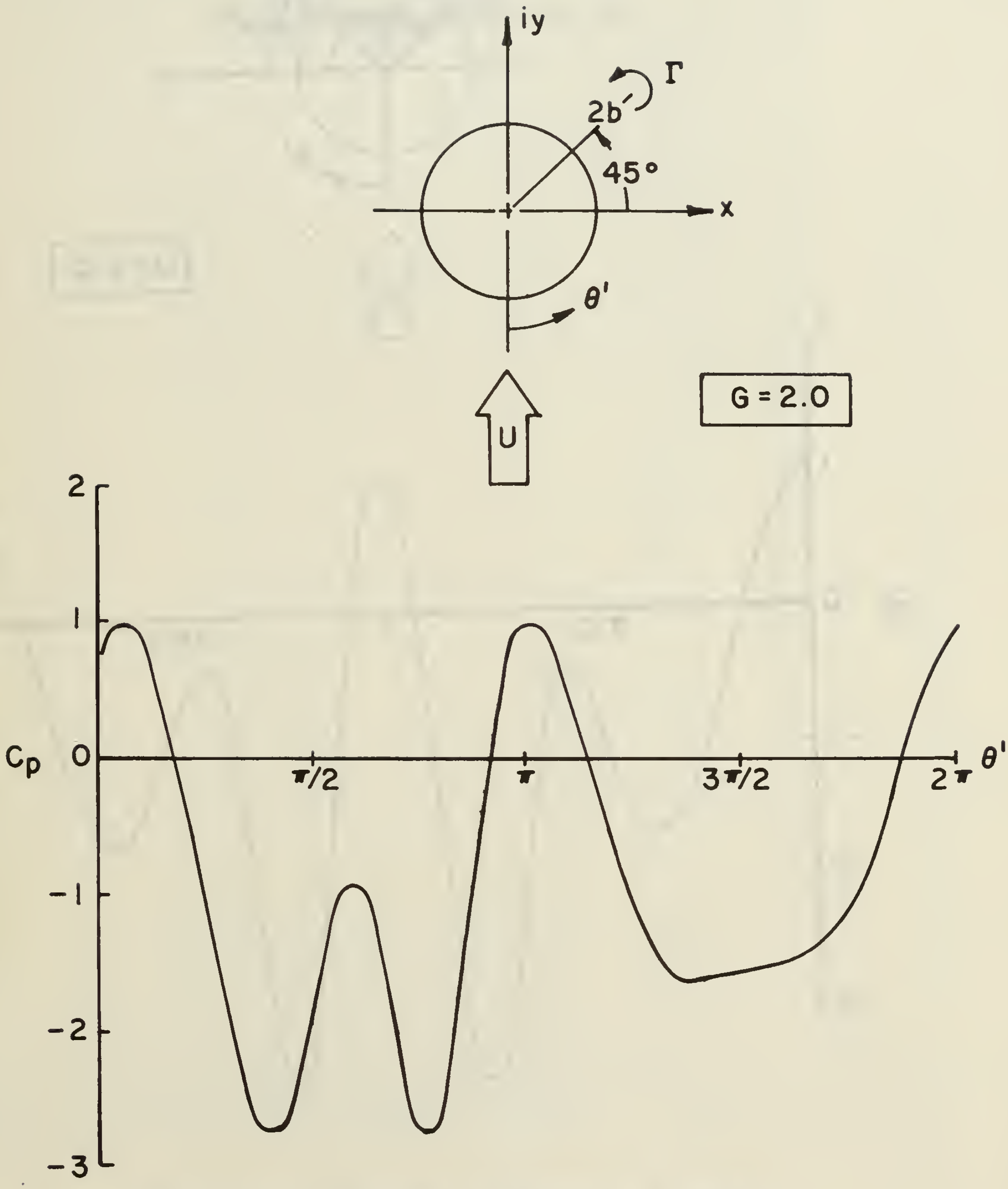

FIGURE 7: THEORETICAL PRESSURE DISTRIBUTION, SINGLE VORTEX CASE

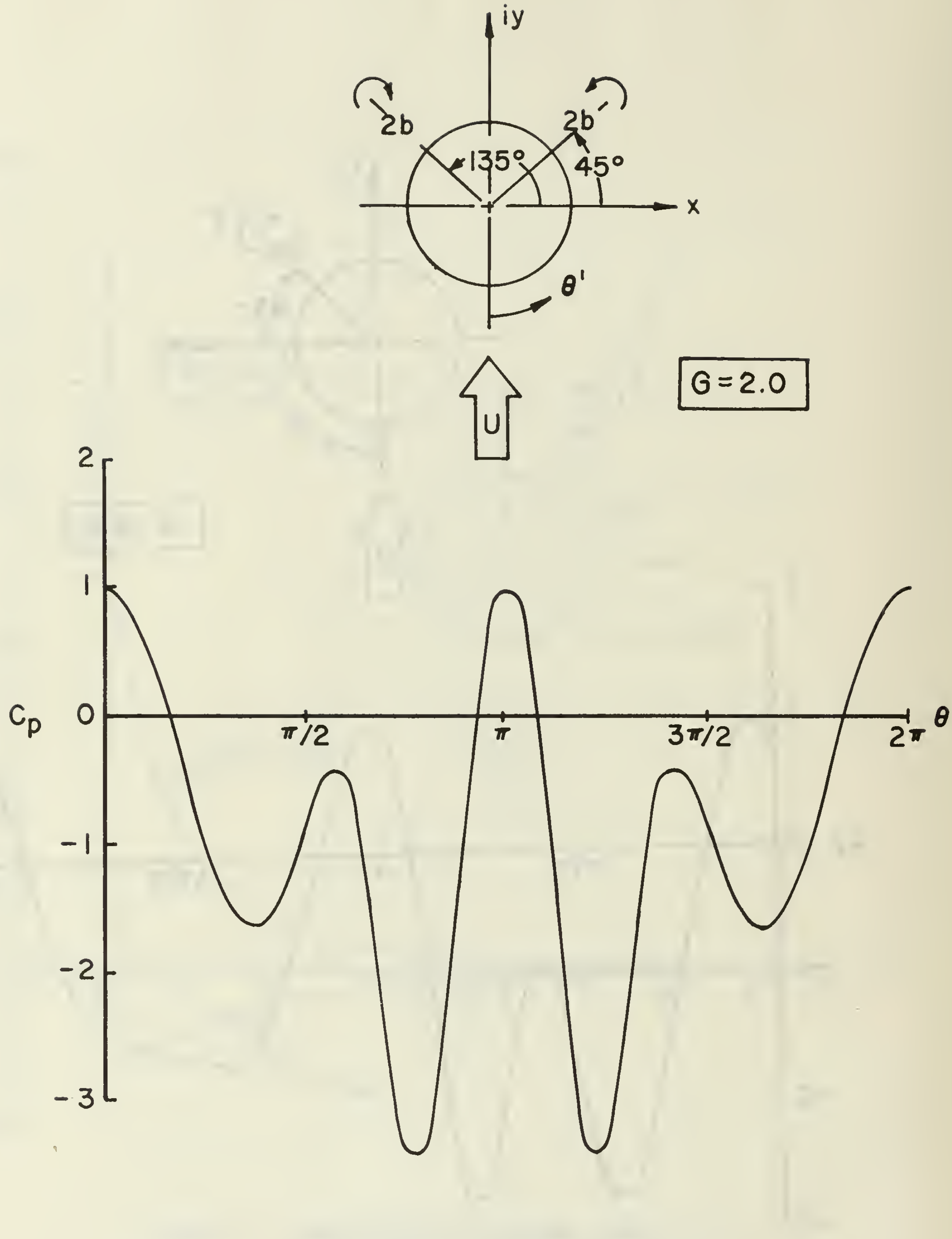

FIGURE 8: THEORETICAL PRESSURE DISTRIBUTION, SYMMETRIC PAIR OF VORTICES

APPENDIX A

FORTRAN PROGRAM LISTING FOR
CALCULATION OF FORCES (Eq. 15)

```
THIS PROGRAM COMPUTES THE NON-DIMENSIONAL FORCE PER UNIT LENGTH
ON A CIRCLE IN THE PRESENCE OF A UNIFORM FLOW AND N VORTICES.

INPUT PARAMETERS:
     NPROB - THE NUMBER OF PROBLEMS TO BE RUN
     N - NUMBER OF VORTICES
     SR(I) - THE REAL PART OF THE RELATIVE POSITION VECTOR OF THE I VORTEX
     SI(I) - THE IMAGINARY PART OF THE RELATIVE POSITION VECTOR OF THE I VORTEX
     G(I) - THE NON-DIMENSIONAL STRENGTH OF THE I VORTEX

OUTPUT PARAMETERS:
     FBAR - THE REAL AND IMAGINARY PARTS OF THE COMPLEX CONJUGATE FORCE
     THETA - THE ANGLE (DEGREES) OF THE FORCE VECTOR MEASURED POSITIVE
             COUNTERCLOCKWISE FROM THE REAL AXIS
     F - THE MAGNITUDE OF THE FORCE


      COMPLEX*8 S,SB,FBAR,SUM1,SUM2
      DIMENSION S(5),SB(5),SR(5),SI(5),G(5)
      DATA PI/3.14159/
      READ(5,900) NPROB
  900 FORMAT(I10)
      DO 100 L=1,NPROB
      WRITE(6,2000)
 2000 FORMAT('1')
      SUM1=CMPLX(0.0,0.0)
      SUM2=CMPLX(0.0,0.0)
      READ(5,1000) N
 1000 FORMAT(I1)
      DO 10 I=1,N
      READ(5,1001) SR(I),SI(I),G(I)
 1001 FORMAT(3F10.6)
      WRITE(6,2001) I,SR(I),SI(I),G(I)
 2001 FORMAT(' ','VORTEX ',I1,/,5X,'SR=',F10.6,/,5X,'SI=',F10.6,/,
     X5X,'G= ',F10.6,//)
      S(I)=CMPLX(SR(I),SI(I))
      SB(I)=CONJG(S(I))
   10 CONTINUE
      DO 20 J=1,N
      SUM1=SUM1+(G(J)/(S(J)*S(J)))
      DO 30 K=1,N
      SUM2=SUM2+((G(J)*G(K))/(S(J)*(S(J)*SB(K)-1.0)))
   30 CONTINUE
   20 CONTINUE
      FBAR=-2.0*SUM1+((2.0*SUM2)/(PI))
      FBARR=REAL(FBAR)
      FBARI=AIMAG(FBAR)
      THET=ATAN2(-FBARI,FBARR)
```

```
      THETA=(THET*180.0)/(PI)
      F=CABS(FBAR)
      WRITE(6,2002) FBARR,FBARI,THETA,F
 2002 FORMAT(' ','FBAR= ',F10.6,5X,F10.6,/,' THETA=',F10.6,/,
     X' F      ',F10.6)
  100 CONTINUE
      STOP
      END
```

APPENDIX B

FORTRAN PROGRAM LISTING FOR CALCULATION OF PRESSURE DISTRIBUTION (Eq. 20)

```
THIS PROGRAM COMPUTES THE PRESSURE COEFFICIENT ON THE BODY FOR A CIRCLE IN
THE PRESENCE OF A UNIFORM FLOW AND N VORTICES

INPUT PARAMETERS:
     N - NUMBER OF VORTICES
     S(I) - RELATIVE POSITION VECTOR OF I VORTEX
     PHI(I) - POLAR ANGLE (RADIANS) OF RELATIVE POSITION VECTOR MEASURED
              POSITIVE COUNTERCLOCKWISE FROM THE REAL AXIS
     G(I) - THE NON-DIMENSIONAL STRENGTH OF THE I VORTEX

OUTPUT PARAMETERS:
     THETA - POLAR ANGLE (RADIANS) MEASURED POSITIVE COUNTERCLOCKWISE
             FROM THE REAL AXIS
     CP - PRESSURE COEFFICIENT
     * - PLOT OF PRESSURE COEFFICIENT VS. THETA


      DIMENSION S(5),PHI(5),G(5),CP(64),THETA(64),D(5)
      REAL LABEL/4H    /
      REAL*8 ITITLE(12)
      DATA PI/3.14159/
      WRITE(6,2000)
 2000 FORMAT('1')
      READ(5,1000) N
 1000 FORMAT(I1)
      IF(N.EQ.0) GO TO 100
      DO 10 I=1,N
      READ(5,1001) S(I),PHI(I),G(I)
 1001 FORMAT(3F10.3)
      WRITE(6,2001) I,S(I),PHI(I),G(I)
 2001 FORMAT(' ','VORTEX ',I1,/,5X,'S=  ',F10.3,/,5X,'PHI=',F10.3,
     X/,5X,'G=  ',F10.3,//)
   10 CONTINUE
  100 WRITE(6,2002)
 2002 FORMAT(' ',6X,'THETA',10X,'CP',//)
      THETA(1)=0.0
      DO 20 J=1,64
      SUM=0.0
      IF(N.EQ.0) GO TO 200
      DO 30 K=1,N
      D(K)=G(K)*((S(K)*COS(THETA(J)-PHI(K))-1.0)/(1.0+(S(K)**2)
     X-2.0*S(K)*COS(THETA(J)-PHI(K))))
      SUM=SUM+D(K)
   30 CONTINUE
  200 CP(J)=-1.0-2.0*COS(2.0*THETA(J))+((8.0*COS(THETA(J))*
     XCOS(2.0*THETA(J)))*(SUM/PI))-(4.0*(SUM**2))/(PI**2)
      WRITE(6,2003) THETA(J),CP(J)
 2003 FORMAT(3X,F10.3,3X,F10.3)
```

```
      THETA(J+1)=THETA(J)+0.1
   20 CONTINUE
      READ(5,1002) (ITITLE(L),L=1,6)
      READ(5,1002) (ITITLE(L),L=7,12)
 1002 FORMAT(6A8)
      CALL DRAW(64,CP,THETA,0,0,LABEL,ITITLE,0,0,0,0,0,0,6,8,1,LAST)
      STOP
      END
```

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13. ABSTRACT

Results of a background investigation and initial theoretical models are described for the flow about axisymmetric bodies at extreme angles of attack. Present state-of-the-art is described with the general conclusions that the current understanding of extreme angle of attack flow phenomena is relatively well developed for angles of attack up to about 40° (symmetric vortex pair in the wake) and that for higher angles of attack there are several important questions yet to be answered. The initial analytical model qualitatively predicts the occurrence of induced yaw forces and is suggested as a basis for further refinements and experimental comparisons. Expressions for the calculation of local forces and pressure distributions are developed for the case of an asymmetric steady vortex wake.

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Flow studies of axisymmetric bodies at extreme angles of attack [5].